Due: 2012/10/24 before class

Homework 5

Problem 1. *Calculate the number of spanning trees on* $[10]$ *where there are two vertices of degree 3, and one vertex of degree 2.*

Problem 2. *Consider the graph* Q_n*, the* n*-dimensional cube for* $n \geq 1$*. Find* $v(Q_n)$*,* $e(Q_n)$*,* $\delta(Q_n)$*, and* $\Delta(Q_n)$*.*

Problem 3. *For any positive integer* n*, define a graph* $G = (V, E)$*, where* V *consists of points in the plane* $(i, 0)$ *for* $i = 0, 1, 2, ..., n+1$*,* $(i, 1)$ *and* $(i, -1)$ *for* $i = 1, 2, ..., n$*. Two vertices are adjacent if (1) their distance in the plane is 1; and (2) one of the vertices is on the* x*-axis. Find the size of* $Aut(G)$*.*

Problem 4. *Prove that, in a connected graph* G*, any two longest paths share a common vertex.*

Problem 5. *Let* G *be a graph on* n *vertices* $(n > 3)$ *with no vertex of degree* $n-1$*. Suppose that for any two vertices of* G*, there is a unique vertex adjacent to both of them.*
(a) If u *and* v *are not adjacent, prove that they have the same degree. (Hint: Construct a bijection between the two sets of neighbors.)*
(b) Show that G *is* k*-regular for some* k*.*
(c) Express n *in terms of* k*.*