

Due: 2012/10/24 before class

Homework 5

Problem 1. Calculate the number of spanning trees on $[10]$ where there are two vertices of degree 3, and one vertex of degree 2.

Problem 2. Consider the graph Q_n , the n -dimensional cube for $n \geq 1$. Find $v(Q_n)$, $e(Q_n)$, $\delta(Q_n)$, and $\Delta(Q_n)$.

Problem 3. For any positive integer n , define a graph $G = (V, E)$, where V consists of points in the plane $(i, 0)$ for $i = 0, 1, 2, \dots, n + 1$, $(i, 1)$ and $(i, -1)$ for $i = 1, 2, \dots, n$. Two vertices are adjacent if (1) their distance in the plane is 1; and (2) one of the vertices is on the x -axis. Find the size of $\text{Aut}(G)$.

Problem 4. Prove that, in a connected graph G , any two longest paths share a common vertex.

Problem 5. Let G be a graph on n vertices ($n > 3$) with no vertex of degree $n-1$. Suppose that for any two vertices of G , there is a unique vertex adjacent to both of them.

(a) If u and v are not adjacent, prove that they have the same degree. (Hint: Construct a bijection between the two sets of neighbors.)

(b) Show that G is k -regular for some k .

(c) Express n in terms of k .