

Due: 2012/10/24 before class

## Homework 5

**Problem 1.** Calculate the number of spanning trees on  $[10]$  where there are two vertices of degree 3, and one vertex of degree 2.

**Problem 2.** Consider the graph  $Q_n$ , the  $n$ -dimensional cube for  $n \geq 1$ . Find  $v(Q_n)$ ,  $e(Q_n)$ ,  $\delta(Q_n)$ , and  $\Delta(Q_n)$ .

**Problem 3.** For any positive integer  $n$ , define a graph  $G = (V, E)$ , where  $V$  consists of points in the plane  $(i, 0)$  for  $i = 0, 1, 2, \dots, n+1$ ,  $(i, 1)$  and  $(i, -1)$  for  $i = 1, 2, \dots, n$ . Two vertices are adjacent if (1) their distance in the plane is 1; and (2) one of the vertices is on the  $x$ -axis. Find the size of  $\text{Aut}(G)$ .

**Problem 4.** Prove that, in a connected graph  $G$ , any two longest paths share a common vertex.

**Problem 5.** Let  $G$  be a graph on  $n$  vertices ( $n > 3$ ) with no vertex of degree  $n-1$ . Suppose that for any two vertices of  $G$ , there is a unique vertex adjacent to both of them.

(a) If  $u$  and  $v$  are not adjacent, prove that they have the same degree. (Hint: Construct a bijection between the two sets of neighbors.)

(b) Show that  $G$  is  $k$ -regular for some  $k$ .

(c) Express  $n$  in terms of  $k$ .