Due: 2012/10/17 before class

Homework 4

Problem 1. *Let n be a positive integer, determine $\sum_{1\le m\le n}\mu(m)\lfloor n/m\rfloor$. Justify your answer.*

Problem 2. *Let $P(k)$ be the number of distinct prime factors of k. e.g., $P(1)=0$, $P(2)=P(8)=P(25)=1$, and $P(30)=3$. Calculate*

$$\sum_{d|20121009}\mu(d)P(\frac{20121009}{d})$$

and justify your answer.

Problem 3. *Let f(n) be the number of integer partitions of n into $x_1+x_2+\ldots+x_t$, where t is any number and $x_i=x_{i+1}+2$ for all $1\le i<t$. Prove that $f(n)$ equals the number of divisors of n that are at most $\sqrt{n}$.*

Problem 4. *(a). Compute $p(n)$ by writing out all the unordered partitions of n, for $n=1,2,3,4,5$.*
(b). Prove $p(n)\le F_n$, where F_n is the n-th Fibonacci number.