

Due: 2012/10/17 before class

## Homework 4

**Problem 1.** Let  $n$  be a positive integer, determine  $\sum_{1 \leq m \leq n} \mu(m) \lfloor n/m \rfloor$ . Justify your answer.

**Problem 2.** Let  $P(k)$  be the number of distinct prime factors of  $k$ . e.g.,  $P(1) = 0$ ,  $P(2) = P(8) = P(25) = 1$ , and  $P(30) = 3$ . Calculate

$$\sum_{d|20121009} \mu(d) P\left(\frac{20121009}{d}\right)$$

and justify your answer.

**Problem 3.** Let  $f(n)$  be the number of integer partitions of  $n$  into  $x_1 + x_2 + \dots + x_t$ , where  $t$  is any number and  $x_i = x_{i+1} + 2$  for all  $1 \leq i < t$ . Prove that  $f(n)$  equals the number of divisors of  $n$  that are at most  $\sqrt{n}$ .

**Problem 4.** (a). Compute  $p(n)$  by writing out all the unordered partitions of  $n$ , for  $n = 1, 2, 3, 4, 5$ .  
(b). Prove  $p(n) \leq F_n$ , where  $F_n$  is the  $n$ -th Fibonacci number.