

Due: 2012/10/17 before class

Homework 4

Problem 1. Let n be a positive integer, determine $\sum_{1 \leq m \leq n} \mu(m) \lfloor n/m \rfloor$. Justify your answer.

Problem 2. Let $P(k)$ be the number of distinct prime factors of k . e.g., $P(1) = 0$, $P(2) = P(8) = P(25) = 1$, and $P(30) = 3$. Calculate

$$\sum_{d|20121009} \mu(d) P\left(\frac{20121009}{d}\right)$$

and justify your answer.

Problem 3. Let $f(n)$ be the number of integer partitions of n into $x_1 + x_2 + \dots + x_t$, where t is any number and $x_i = x_{i+1} + 2$ for all $1 \leq i < t$. Prove that $f(n)$ equals the number of divisors of n that are at most \sqrt{n} .

Problem 4. (a). Compute $p(n)$ by writing out all the unordered partitions of n , for $n = 1, 2, 3, 4, 5$.

(b). Prove $p(n) \leq F_n$, where F_n is the n -th Fibonacci number.