Due: 2012/10/10 before class

Homework 3

Problem 1. *A matrix is* good *if in each row there are no repeated elements, and in each column there are no repeated elements. How many good matrices are there with* 2 *rows and* 6 *columns whose elements are all from* $[6]$*?*

Problem 2. *For each* n*, find the simple form for* $S(n, n-2)$*,* $S(n, n-3)$*,* $s(n, n-2)$*, and* $s(n, n-3)$*.*

Problem 3. *Prove that the Stirling numbers of the 1st kind* $s(n, k)$ *is an even number whenever* $2k < n$ *and* $n > 0$*.*

Problem 4. *Count the number of permutations* $x_1, x_2, \cdots, x_{2n}$ *of* $[2n]$ *such that* $x_i + x_{i+1} \neq 2n+1$ *for all* $1 \leq i \leq 2n-1$*.*

Problem 5. *Suppose* $n > 4$ *and there is an array of numbers* $a_1, a_2, ..., a_n$*, where each* a_i *is either* 1 *or* -1*. If*

$$a_1a_2a_3a_4 + a_2a_3a_4a_5 + ... + a_{n-1}a_na_1a_2 + a_na_1a_2a_3 = 0,$$

prove that n *is a multiple of 4.*

Problem 6. *In an* $n \times n$ *matrix each entry is filled with an integer. In each step you can select one line (a row or a column) and change the sign of all the* n *numbers in that line.*
Prove that in finite number of steps, one can change it to a matrix with non-negative sums on every line.