Due: 2012/10/10 before class

Homework 3

Problem 1. *A matrix is* good *if in each row there are no repeated elements, and in each column there are no repeated elements. How many good matrices are there with* 2 *rows and* 6 *columns whose elements are all from* [6]*?*

Problem 2. *For each n, find the simple form for $S(n, n-2)$, $S(n, n-3)$, $s(n, n-2)$, and $s(n, n-3)$.*

Problem 3. *Prove that the Stirling numbers of the 1st kind $s(n, k)$ is an even number whenever $2k < n$ and $n > 0$.*

Problem 4. *Count the number of permutations $x_1, x_2, \cdots, x_{2n}$ of $[2n]$ such that $x_i + x_{i+1} \neq 2n + 1$ for all $1 \leq i \leq 2n - 1$.*

Problem 5. *Suppose $n > 4$ and there is an array of numbers $a_1, a_2, ..., a_n$, where each a_i is either* 1 *or* -1. *If*

$$a_1a_2a_3a_4 + a_2a_3a_4a_5 + ... + a_{n-1}a_na_1a_2 + a_na_1a_2a_3 = 0,$$

prove that n is a multiple of 4.

Problem 6. *In an $n \times n$ matrix each entry is filled with an integer. In each step you can select one line (a row or a column) and change the sign of all the n numbers in that line.*
Prove that in finite number of steps, one can change it to a matrix with non-negative sums on every line.