Due: 2012/09/26 before class

Homework 2

Problem 1. *We randomly generate numbers $a_1, a_2, ..., a_n$ such that each a_i is uniformly and independently sampled from $[100]$. What is the probabilty that $\prod_1^n a_i$ is a multiple of 10?*

Problem 2. *Draw the Hasse diagram for the divisibility lattice $([12], |)$.*

Problem 3. *Let n be a positive integer, find*

$$\sum_{i=1}^{n} (-1)^{i-1} i \binom{n}{i}.$$

Justify your answer.

Problem 4. *Give a combinatorial proof for the following equation. For any positive integers a and b,*

$$\sum_{i=0}^{a} \binom{a}{i} \binom{b+i}{a} = \sum_{i=0}^{a} \binom{a}{i} \binom{b}{i} 2^i.$$

Problem 5. *Let $\mathcal{L} = (X, \preccurlyeq)$ be a finite lattice. For any $X_1, X_2 \subseteq X$, define*

$$\mu(X_1, X_2) = \sum_{x_1 \in X_1, x_2 \in X_2} \mu(x_1, x_2).$$

Suppose A, B, and C is a partition of X such that A is an ideal and C is a filter. Prove that

$$\mu(A, C) = \mu(B, B) - 1.$$