Do not need to hand in

Homework 14

Problem 1. *Let A be the adjacency matrix of the complete bipartite graph $K_{3,3}$, compute the eigenvalues of A.*

Problem 2. *Let n be a positive integer. Consider*

$$S = \{(x, y, z) : x, y, z \in [n] \cup \{0\}, x + y + z > 0\}$$

as a set of $(n+1)^3 - 1$ points in $\mathbf{R}^3$. We proved that $3n$ planes are needed to touch all the points if none of the planes is allowed to touch $(0, 0, 0)$. Shorten the proof using Combinatorial Nullstellensatz.

Problem 3. *Let A_1, A_2, ..., A_m be m distinct subsets of $[n]$ such that each $|A_i|$ is even and for any $i \neq j$, $|A_i \cap A_j|$ is odd. How big can m be? Prove your answer. (i.e., Prove the bound, and show examples where the bound is reached.)*

Problem 4. *Prove that, for any n, there is a two-distance set of size at least $\binom{n+1}{2}$ in $\mathbf{R}^n$.*

As usual, the last problem is always a little harder than usual.

Problem 5. *(extra) Prove that, any two-distance set in $\mathbf{R}^n$ has size at most $\binom{n+2}{2}$.*

Hint. This is $n + 1$ less than the bound we proved in class. Try to add $n + 1$ new polynomials to the polynomials we used, and show that they are still linearly independent.