Do not need to hand in

Homework 13

Problem 1. *Let* $f : \mathbf{R}^2 \to [3]$ *be a 3-colouring of the plane. Prove that there will be two points* a *and* b*, such that* $|ab| = 1$*, and* $f(a) = f(b)$*.*

Problem 2. *Let* $2 \leq k \leq \sqrt{n}$*. Consider a set* S *of* n *points in the plane. A line is called* good *if it contains at least* k *points from* S*. Prove that there are at most* cn^2/k^3 *good lines, where* c *is a constant independent of* n *and* k*.*

Problem 3. *The* L_1 *distance for two points in the plane is*

$$d((x, y), (x', y')) = |x - x'| + |y - y'|.$$

Let $n \geq 2$ *and* V *be the set of points in the plane*

$$V = \{(1, 3), (2, 4), (3, 1), (4, 2)\} \cup \{(n + i, 2n - i) : i = 0, 1, 2, ..., n\}.$$

And let the hypergraph $\mathcal{F}$ *be the one with the edges*

$$\{\{a, b, c\} \subseteq V : |\{a, b, c\}| = 3, d(a, b) + d(b, c) = d(a, c)\}.$$

How many different lines are there in this system?

Problem 4. *Let* $\mathcal{F}$ *be a 3-uniform hypergraph on* $[n]$ *where among any four points there are less than 4 hyperedges. Prove that in this system there is a line equal to* $[n]$*, or there are at least* n *different lines.*