Homework 12

Problem 1. *Let n be a positive integer and $\mathcal{F}$ be an intersecting family in $2^{[n]}$ and $|\mathcal{F}| = 2^{n-1}$. Prove that $\mathcal{F}$ is a filter. (i.e., $B \in \mathcal{F}$ whenever there is some $A \in \mathcal{F}$ and $A \subseteq B$.)*

Problem 2. *Describe a family $\mathcal{F}$ of* 120 *sets in $2^{[12]}$, where each set is of size 5, and any two sets $A, B \in \mathcal{F}$ have $|A \cap B| \geq 2$.*

Problem 3. *Describe a family $\mathcal{F}$ of* more than 30 *sets in $2^{[11]}$, where each set is of size 5, and any two sets $A, B \in \mathcal{F}$ have $|A \cap B| \geq 3$.*

Problem 4. *Given n real numbers a_1, ..., a_n such that each $|a_i| \geq 1$. For any subset $A \subseteq [n]$, define $S(A) = \sum_{i \in A} a_i$. If A_1, ..., A_m are m distinct subsets of $[n]$ such that $|S(A_i) - S(A_j)| < 1$ for any $i \neq j$, how big can m be? Justify your answer. (i.e., For each n, find a number $f(n)$, prove that that we always have $m \leq f(n)$, and show an example where $m = f(n)$.)*