

## Homework 12

**Problem 1.** Let  $n$  be a positive integer and  $\mathcal{F}$  be an intersecting family in  $2^{[n]}$  and  $|\mathcal{F}| = 2^{n-1}$ . Prove that  $\mathcal{F}$  is a filter. (i.e.,  $B \in \mathcal{F}$  whenever there is some  $A \in \mathcal{F}$  and  $A \subseteq B$ .)

**Problem 2.** Describe a family  $\mathcal{F}$  of 120 sets in  $2^{[12]}$ , where each set is of size 5, and any two sets  $A, B \in \mathcal{F}$  have  $|A \cap B| \geq 2$ .

**Problem 3.** Describe a family  $\mathcal{F}$  of more than 30 sets in  $2^{[11]}$ , where each set is of size 5, and any two sets  $A, B \in \mathcal{F}$  have  $|A \cap B| \geq 3$ .

**Problem 4.** Given  $n$  real numbers  $a_1, \dots, a_n$  such that each  $|a_i| \geq 1$ . For any subset  $A \subseteq [n]$ , define  $S(A) = \sum_{i \in A} a_i$ . If  $A_1, \dots, A_m$  are  $m$  distinct subsets of  $[n]$  such that  $|S(A_i) - S(A_j)| < 1$  for any  $i \neq j$ , how big can  $m$  be? Justify your answer. (i.e., For each  $n$ , find a number  $f(n)$ , prove that that we always have  $m \leq f(n)$ , and show an example where  $m = f(n)$ .)