Due: 2011/12/05 before class

Homework 11

Problem 1. *Let $\mathcal{A}$ be a set of $(r+1)$-subsets of $[n]$, i.e., $\mathcal{A} \subseteq \binom{[n]}{r+1}$. Its* shadow *is defined as*

$$\partial\mathcal{A} = \{B \in \binom{[n]}{r} : \exists A \in \mathcal{A}, B \subseteq A.\}$$

Prove that the proportion of $\partial\mathcal{A}$ in its level is at least as big as the proportion of $\mathcal{A}$ in level $(r+1)$, i.e.,

$$\frac{\partial\mathcal{A}}{\binom{n}{r}} \geq \frac{\mathcal{A}}{\binom{n}{r+1}}.$$

Problem 2. *A chain of sets $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_t$ is n-symmetric if $|A_{i+1}| = |A_i| + 1$ for any $1 \leq i < t$, and $|A_1| + |A_t| = n$.*
Partition the poset $2^{[4]}$ into 4-symmetric chains.
(Think about this: Prove that we can partition $2^{[n]}$ into n-symmetric chains.)

Problem 3. *Let $n = 10$, consider subsets of $[n]$.*
(a) Are there 200 subsets $(A_1, A_2, \ldots, A_{200})$ such that $A_i \not\subseteq A_j$ for any $i \neq j$?
(b) Are there 100 subsets $(B_1, B_2, \ldots, B_{100})$ such that $|B_i| = 5$ for all i, and $B_i \cap B_j \neq \emptyset$ for all $i < j$?

Problem 4. *A matrix with integer entries is called good if no row nor column contains the same number twice.*
Let A be a $k \times n$ good matrix ($k < n$) with entries from $[n]$, prove that there is an $n \times n$ good matrix B with entries from $[n]$ such that the first k rows of B is identical to A.