

Due: 2011/12/05 before class

## Homework 11

**Problem 1.** Let  $\mathcal{A}$  be a set of  $(r+1)$ -subsets of  $[n]$ , i.e.,  $\mathcal{A} \subseteq \binom{[n]}{r+1}$ . Its shadow is defined as

$$\partial\mathcal{A} = \{B \in \binom{[n]}{r} : \exists A \in \mathcal{A}, B \subseteq A.\}$$

Prove that the proportion of  $\partial\mathcal{A}$  in its level is at least as big as the proportion of  $\mathcal{A}$  in level  $(r+1)$ , i.e.,

$$\frac{|\partial\mathcal{A}|}{\binom{n}{r}} \geq \frac{|\mathcal{A}|}{\binom{n}{r+1}}.$$

**Problem 2.** A chain of sets  $A_1 \subseteq A_2 \subseteq \dots \subseteq A_t$  is  $n$ -symmetric if  $|A_{i+1}| = |A_i| + 1$  for any  $1 \leq i < t$ , and  $|A_1| + |A_t| = n$ .

Partition the poset  $2^{[4]}$  into 4-symmetric chains.

(Think about this: Prove that we can partition  $2^{[n]}$  into  $n$ -symmetric chains.)

**Problem 3.** Let  $n = 10$ , consider subsets of  $[n]$ .

(a) Are there 200 subsets  $(A_1, A_2, \dots, A_{200})$  such that  $A_i \not\subseteq A_j$  for any  $i \neq j$ ?

(b) Are there 100 subsets  $(B_1, B_2, \dots, B_{100})$  such that  $|B_i| = 5$  for all  $i$ , and  $B_i \cap B_j \neq \emptyset$  for all  $i < j$ ?

**Problem 4.** A matrix with integer entries is called good if no row nor column contains the same number twice.

Let  $A$  be a  $k \times n$  good matrix ( $k < n$ ) with entries from  $[n]$ , prove that there is an  $n \times n$  good matrix  $B$  with entries from  $[n]$  such that the first  $k$  rows of  $B$  is identical to  $A$ .