Due: 2012/9/19 before class

Homework 1

Problem 1. *Calculate the following:*
(a) $\binom{10}{8}$*;*
(b) The inverse of $1, 2, 3, 4, 5, 6$ *in* $\mathbf{Z}_7$*, i.e. for each* $1 \leq i \leq 6$*, find* $j = i^{-1}$ *such that* $ij \equiv 1 \mod 7$*;*
(c) The rightmost digit in the decimal $\binom{449}{137}$*, i.e.* $\binom{449}{137} \mod 10$*.*

Problem 2. *Find the number of ordered pairs* (A, B) *such that* $A, B \subseteq [n]$ *and* $A \cap B = \emptyset$*.*

Problem 3. *Prove*

$$\sum_{r=0}^{n} r^2 \binom{n}{r} = n(n+1)2^{n-2}.$$

For extra challenge, find a combinatorial proof.

Problem 4. *Consider 20 blue segments* $(0, i)$*-*$(9, i)$ *and* $(i, 0)$*-*$(i, 9)$ *for* $i = 0, 1, ..., 9$*. They form a matrix of* 9×9 *unit squares. A square is blue if it has four blue edges.*
(a) How many blue squares can you find in such a picture?
(b) How many pairs of blue squares (A, B) *can you find such that* A *and* B *are disjoint? (Two squares are disjoint if they do not share any interior points.)*

Problem 5. *Use the* 9×9 *unit squares again. If each unit square is filled with a distinct number from* $[81]$*, prove that there are always two neighboring squares (vertically or horizontally adjacent) with difference at least 9.*