Due: 2011/10/26 before class

Homework 5

Problem 1. *Given two graphs G and H, where $V(G)$ and $V(H)$ are disjoint, their* join *is defined to be the graph with a copy of G, a copy of H and additional edges joining all the pairs between G and H. Formally,*

$$G \vee H = (V(G) \cup V(H), E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}).$$

Clearly, $K_3 \vee C_5$ does not contain a copy of K_6. Prove that, if its edges are colored Y and B, there is always a monochromatic K_3.

Problem 2. *For each n, define the set of points in the plane*

$$A_n = \{(x, y) : x, y \in \mathbf{Z}, 1 \leq x, y \leq n\}.$$

Prove that, for any integer $c > 0$, there exists $N = N(c)$, such that no matter how we c-color A_N, there are 4 points in A_N with the same color and form an axis-parallel rectangle.

Problem 3. *For any positive integers c and r, there exists $N = N(c, r)$, such that no matter how we c-color $[N]$, there are $r + 1$ numbers x, y_1, y_2, ..., y_r such that all the 2^r sums*

$$x + \sum_{i \in S} y_i : S \subseteq [r]$$

are not bigger than N, and have the same color. (This lemma is perhaps the first Ramsey type result in the hisotry, due to David Hilbert, 1892.)

Problem 4. *Prove the stronger version of Schur's theorem: For any positive integer c, there exists $S^*(c)$ such that no matter how we color $[S^*(c)]$ by c colors, there are* distinct *$x, y, z \in [S(c)]$ of the same color such that $x + y = z$. (Hint: Prove that $S^*(c) \leq N_{2c}(3; 2)$.)*