Due: 2011/10/19 before class

Homework 4

Problem 1. *Prove that, if G is a k-regular ($k \geq 1$) bipartite graph with bipartition A and B, then $|A| = |B|$.*

Problem 2. *For each n, find $n+1$ distinct subsets of $[n]$, A_i ($1 \leq i \leq n+1$) such that for each $x \in [n]$, there are $1 \leq i < j \leq n+1$ where $A_i - x = A_j - x$.*

Problem 3. *Prove that, in a connected graph G, any 2 longest paths share a common vertex. (Note: write your proof as convincing as possible.)*

Problem 4. *Let T be a tree, T_i ($1 \leq i \leq k$) be subtrees of T. Prove that, if any two of the T_i's share a common vertex, then all the T_i's share a common vertex.*

Problem 5. *For any positive integer n, define a graph $G = (V, E)$, where V consists of points in the plane $(i, 0)$ for $i = 0, 1, 2, ..., n+1$, $(i, 1)$ and $(i, -1)$ for $i = 1, 2, ..., n$. Two vertices are adjacent if their distance in the plane is 1. Find the size of $Aut(G)$.*

Problem 6. *For each $n > 5$, find a graph G of order n for which $|Aut(G)| = 1$.*

Problem 7. *Let G be a graph on n vertices ($n > 3$) with no vertex of degree $n-1$. Suppose that for any two vertices of G, there is a unique vertex adjacent to both of them.*
(a) If u and v are not adjacent, prove that they have the same degree. (Hint: Construct a bijection between the two sets of neighbors.)
(b) Show that G is k-regular for some k.
(c) Express n in terms of k.