Due: 2011/10/12 before class

Homework 3

Problem 1. *For each of the following sub-problem, find an answer and justify.*

(a) For any non-negative integer n, how many solutions (x, y) are there such that x and y are non-negative integers and $x + 2y = n$?

(b) For any non-negative integer n, how many solutions (x, y) are there such that x and y are non-negative integers and $2x + 3y = n$?

Problem 2. *For a non-negative integer n, let A_n be the number of non-negative integer pairs (x, y) where one of the following is satisfied:*

$$x+2y = n, 2x+3y = n-1, 3x+4y = n-2, ..., nx+(n+1)y = 1, (n+1)x+(n+2)y = 0.$$

Compute A_n. (You may find the following approach useful: Find the generating function for $x + 2y = n$, and then find the generating function for A_n, and simplify it.)

Problem 3. *For each n, find the simple form for $S(n, n-2)$, $S(n, n-3)$, $s(n, n-2)$, and $s(n, n-3)$.*

Problem 4. *Prove that the Stirling numbers of the 1st kind $s(n, k)$ is an even number whenever $2k < n$ and $n > 0$.*