

Due: 2011/10/12 before class

## Homework 3

**Problem 1.** For each of the following sub-problem, find an answer and justify.

(a) For any non-negative integer  $n$ , how many solutions  $(x, y)$  are there such that  $x$  and  $y$  are non-negative integers and  $x + 2y = n$ ?

(b) For any non-negative integer  $n$ , how many solutions  $(x, y)$  are there such that  $x$  and  $y$  are non-negative integers and  $2x + 3y = n$ ?

**Problem 2.** For a non-negative integer  $n$ , let  $A_n$  be the number of non-negative integer pairs  $(x, y)$  where one of the following is satisfied:

$$x+2y = n, 2x+3y = n-1, 3x+4y = n-2, \dots, nx+(n+1)y = 1, (n+1)x+(n+2)y = 0.$$

Compute  $A_n$ . (You may find the following approach useful: Find the generating function for  $x + 2y = n$ , and then find the generating function for  $A_n$ , and simplify it.)

**Problem 3.** For each  $n$ , find the simple form for  $S(n, n - 2)$ ,  $S(n, n - 3)$ ,  $s(n, n - 2)$ , and  $s(n, n - 3)$ .

**Problem 4.** Prove that the Stirling numbers of the 1st kind  $s(n, k)$  is an even number whenever  $2k < n$  and  $n > 0$ .