Due: 2011/12/07 before class

Homework 10

The first problem is related to the topic we will cover in the next week.

Problem 1. *Let $n = 10$, consider subsets of $[n]$.*
(a) Are there 200 subsets $(A_1, A_2, ..., A_{200})$ such that $A_i \not\subseteq A_j$ for any $i \neq j$?
(b) Are there 100 subsets $(B_1, B_2, ..., B_{100})$ such that $|B_i| = 5$ for all i, and $B_i \cap B_j \neq \emptyset$ for all $i < j$?

The second problem says that, in order to check the regularity of a pair, it is enough to check the subsets of size just above the threshold.

Problem 2. *Let $G = (V, E)$ be a graph and (X, Y) be a pair of nonempty disjoint subset of V. Let ϵ be a constant such that $1 > \epsilon > 0$. Let $t_1 = \lceil \epsilon |X| \rceil$ and $t_2 = \lceil \epsilon |Y| \rceil$. Prove that, if $|d(A, B) - d(X, Y)| < \epsilon$ for any pair (A, B) where $A \in \binom{X}{t_1}$ and $B \in \binom{Y}{t_2}$, then (X, Y) is ϵ-regular.*

The last problem is a weaker version of Erdös-Sós conjecture.

Problem 3. *Let G be a graph with n vertices and at least $(k-1)n$ edges.*
(a). Prove that G has a subgraph G' where $\delta(G') \geq k$.
(b). Prove that G contains every tree with k edges.