

## Homework 10

The first problem is related to the topic we will cover in the next week.

**Problem 1.** Let  $n = 10$ , consider subsets of  $[n]$ .

- (a) Are there 200 subsets  $(A_1, A_2, \dots, A_{200})$  such that  $A_i \not\subseteq A_j$  for any  $i \neq j$ ?
- (b) Are there 100 subsets  $(B_1, B_2, \dots, B_{100})$  such that  $|B_i| = 5$  for all  $i$ , and  $B_i \cap B_j \neq \emptyset$  for all  $i < j$ ?

The second problem says that, in order to check the regularity of a pair, it is enough to check the subsets of size just above the threshold.

**Problem 2.** Let  $G = (V, E)$  be a graph and  $(X, Y)$  be a pair of nonempty disjoint subset of  $V$ . Let  $\epsilon$  be a constant such that  $1 > \epsilon > 0$ . Let  $t_1 = \lceil \epsilon|X| \rceil$  and  $t_2 = \lceil \epsilon|Y| \rceil$ . Prove that, if  $|d(A, B) - d(X, Y)| < \epsilon$  for any pair  $(A, B)$  where  $A \in \binom{X}{t_1}$  and  $B \in \binom{Y}{t_2}$ , then  $(X, Y)$  is  $\epsilon$ -regular.

The last problem is a weaker version of Erdős-Sós conjecture.

**Problem 3.** Let  $G$  be a graph with  $n$  vertices and at least  $(k - 1)n$  edges.

- (a). Prove that  $G$  has a subgraph  $G'$  where  $\delta(G') \geq k$ .
- (b). Prove that  $G$  contains every tree with  $k$  edges.