Due: 2011/09/19 before class

Homework 1

Problem 1. *Calculate the following:*
(a) $\binom{10}{8}$*;*
(b) The inverse of $1, 2, 3, 4, 5, 6$ *in* $\mathbf{Z}_7$*, i.e. for each* $1 \le i \le 6$*, find* $j = i^{-1}$ *such that* $ij \equiv 1 \mod 7$*;*
(c) The rightmost digit in the decimal $\binom{449}{137}$*, i.e.* $\binom{449}{137} \mod 10$*.*

Problem 2. *Find the number of ordered pairs* (A, B) *such that* $A, B \subseteq [n]$ *and* $A \cap B = \emptyset$*.*

Problem 3. *Prove*

$$\sum_{r=0}^{n} r^2 \binom{n}{r} = n(n+1)2^{n-2}.$$

For extra challenge, find a combinatorial proof.

Problem 4. *Give a combinatorial proof for*

$$\sum_{k=0}^{n} \binom{2k}{k} \binom{2n-2k}{n-k} = 4^n.$$